(PWL) is proportional to M^5 for standing or slowly moving wave patterns and $M^{2,3}$ for the convected patterns with characteristic speeds equal to that of the turbulent eddies.

The effect of reflections on the radiation intensity is not well understood yet. Panels of various lengths are being investigated presently in order to clarify this question.

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Nonequilibrium Electric Conductivity of Two-Phase Metal Vapors

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RANKINE cycle nuclear-electric power systems, with alkali metal working fluids and MHD generators used in place of turbines, show promise of producing considerably higher powers per unit of system mass than seem possible with turboalternators. A key problem in the development of such systems is the attainment of adequate electric conductivity in the alkali metals at the temperatures accessible to reactors.

Although it appears possible to attain high conductivities in seeded noble gases by utilizing the tendency for the electron temperature to be elevated by Joule heating, it has not been clear whether this effect would yield high conductivities in wet alkali-metal vapors.

The intrinsic difference is that formerly the free electrons and valence electrons formed an essentially closed thermodynamic system that is weakly coupled to the translational degrees of freedom of the gas, whereas in the present case the electron gas may be rather strongly coupled to the liquid phase through the processes of absorption and re-emission of the electrons by droplets of liquid.

An approximate theory of the wet nonequilibrium gas has been developed. Its principal features will be indicated.

The theory has been tested by comparison with experiments conducted in a high-temperature (up to 2000°K) potassium loop, whose principal features will also be indicated. More complete descriptions of both theory and experiment are given in Ref. 1.

Theory

Consider a plasma in which all droplets are of the same size, but have varying charges Z. If the probability per unit time that a drop will change from charge Z to $Z - 1$ is denoted $\alpha_{Z,Z-1}$, the requirement for a steady population is as follows:

$$N_{Z-1}/N_Z = \alpha_{Z,Z-1}/\alpha_{Z-1,Z}$$

where N_Z is the number density of drops of charge Z.

A detailed development including electron and ion capture and thermionic emission, under the assumption of a Maxwellian electron and energy distribution, leads to the following expressions for the α:

$$\left.\begin{aligned}\alpha_{Z,Z-1} &= N_e\left(\frac{kT_e}{2\pi m_e}\right)^{1/2}\left(1+\frac{W_Z-\phi_s}{kT_e}\right)\\ \alpha_{Z-1,Z} &= \frac{4\pi m_e}{h^3}(kT)^2e^{-W_Z/kT}\left(1+\frac{W_Z-\phi_s}{kT}\right)+\\ &\qquad N_i\left(\frac{kT}{2\pi m_a}\right)^{1/2}e^{-V_i/kT}\end{aligned}\right\}Z\geq 1$$

$$\left.\begin{aligned}\alpha_{Z,Z-1} &= N_e\left(\frac{kT_e}{2\pi m_e}\right)^{1/2}e^{(W_Z-\phi_s)/kT_e}\\ \alpha_{Z-1,Z} &= \frac{4\pi m_e}{h^3}(kT)^2\,e^{-\phi_s/kT}+\\ &\qquad N_i\left(\frac{kT}{2\pi m_a}\right)^{1/2}\left(1-\frac{V_i}{kT}\right)\end{aligned}\right\}Z\leq 0$$

where

$$W_Z = \phi_s + [\tfrac{3}{8} + Z - 1](e^2/4\pi\epsilon_0 A)$$

$$Vi = (Z - \tfrac{3}{8})\, e^2/4\pi\epsilon_0 A$$

Here, A is the droplet radius, ϕ_s is the work function of the flat liquid surface, and the rest of the notation is conventional.

The resultant expressions for N_{Z-1}/N_Z plus Saha's equation for the atomic ionization, together with mass and charge conservation conditions, have been used in a computer program to study the ionization of wet potassium vapor as a function of electron temperature. Figure 1 indicates the essential results. Although the equilibrium ionization depends on the droplet size, over a considerable range (A greater than 50 Å), it is well approximated by

$$N_e = 2\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\phi_s/kT}$$

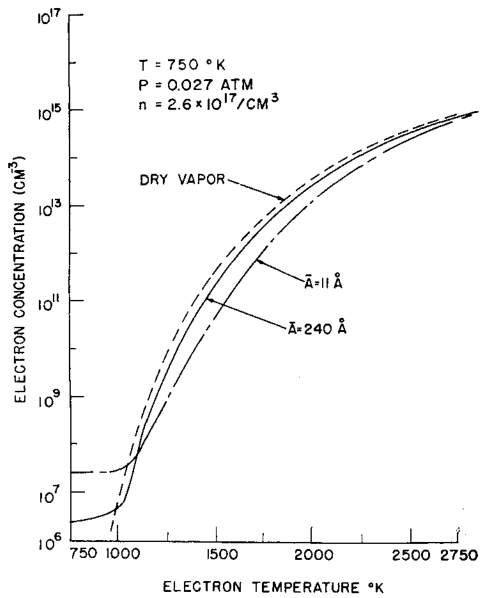

Fig. 1 Nonequilibrium ionization of potassium vapor as a function of electron temperature T_e for gas temperature T = 750°K.

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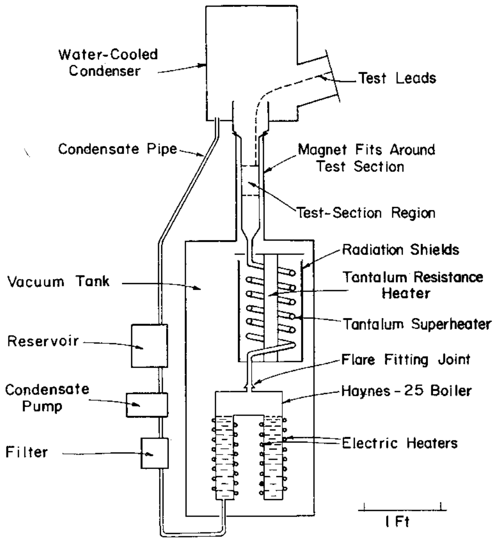

Fig. 2 Potassium-vapor generator.

which is the electron density in equilibrium with a flat surface of the droplet material. Droplets of radius greater than 200 Å have very little effect on the highly nonequilibrium ionization ($T_e - T$ greater than 300°K). Smaller droplets reduce the vapor ionization level significantly.

By equating the Joule heating in the fluid to the rate of energy loss from the electrons to the other constituents, the degree of electron heating can be determined.[2] In this instance, through their large inelastic cross sections, droplets prove to be a more serious hindrance to the achievement of nonequilibrium ionization. Assuming elastic electron-atom and electron-ion collisions, we have

$$\mathbf{j}\cdot\mathbf{E} = 2\,(m_e/m_a)\,[\tfrac{3}{2}k(T_e - T)]N_e\nu_a + \tfrac{3}{2}k(T_e - T)N_e\nu_d$$

or

$$T_e - T = \frac{2\,\mathbf{j}\cdot\mathbf{E}m_a}{3kN_e m_e \nu_a \delta_{\text{eff}}}$$

where

$$\delta_{\text{eff}} = 2[1 + (m_a\nu_d/2m_e\nu_a)]$$

and ν_a and ν_d are the collision frequencies with atoms and drops, respectively.

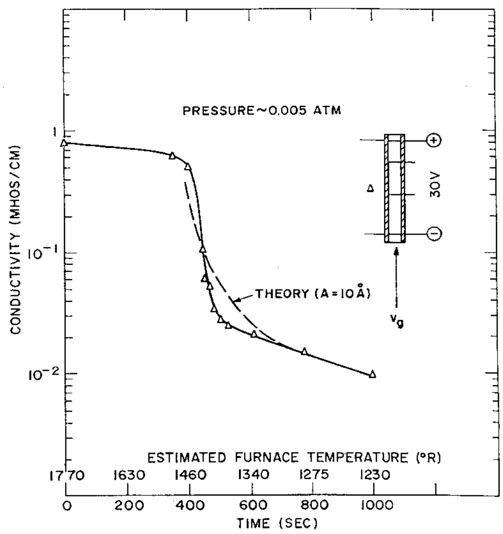

Fig. 3 Effective over-all nonequilibrium conductivity measured during cooling of superheater.

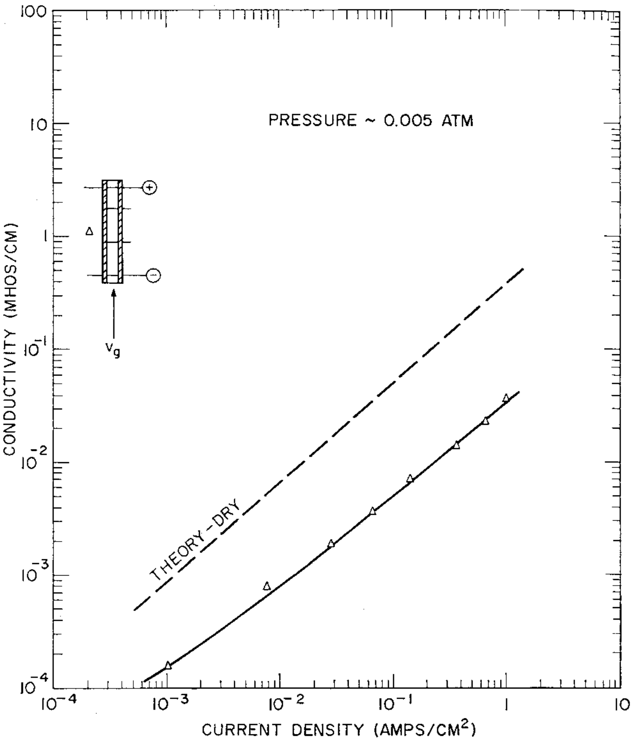

Fig. 4 Effective nonequilibrium conductivity of wet potassium vapor and theoretical characteristics of a dry vapor at the same pressure and temperature.

Computation of ν_d requires a knowledge of N_Z, since the droplet inelastic cross-sections are sensitive to their charge. When ($T_e - T$) is greater than 300°K, most droplets are negatively charged, and because of this $\delta_{\text{eff}} \to 2$. An important exception holds for very small droplets (A less than 50 Å), which are predominantly neutral and can therefore cause large values of δ_{eff}. Thus, in general, the nonequilibrium Ohm's law of a two-phase vapor should differ little from that of the dry vapor unless the droplets are very small.

Experiments

The high-temperature potassium loop is shown schematically in Fig. 2. During the experiments reported here, the test section constructed of alumina was mounted in the condenser at the end of the testing region. The test section Mach number was 0.6, which is sufficient to produce approximately 2% condensation if the gas is saturated at stagnation conditions. By superheating, the wetness could then be varied.

This loop has been operated for many hours and at temperatures up to 1850°K. Its design mass flow rate, at a stagnation pressure of 1 atm, is 10 g/sec.

Figure 3 shows the results of an experiment in which the superheater temperature was varied, thereby varying the wetness. The rapid decrease of conductivity with increasing wetness (decreasing superheat) is clearly evident.

The theoretical curve is given for a droplet radius of 10 Å. Since the depression in conductivity is very sensitive to this radius, it is certain that the droplets were, on the average, very small. In fact, the 10 Å is roughly the nucleation size for potassium.

A nonequilibrium characteristic for the wet plasma is shown in Fig. 4. It indicates a δ_{eff} of approximately 100, which is consistent with the 10 Å drop size.

Conclusions

These results, although tentative, suggest first that nonequilibrium ionization may be difficult to attain in alkali-metal MHD generators with wet vapor used, unless droplet growth can be stimulated; second, electrical techniques such as this one may be useful for study of droplet nucleation and growth; and third, the successful operation of the high-temperature loop is an indication that alkali-metal systems may be operable at somewhat higher temperatures than are now proposed.

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Stability of Damped Mechanical Systems

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The stability of damped, mechanical systems is of great interest in space dynamics. The main observation of this note is that for some systems the Hamiltonian function of mechanics is a very useful "testing function" for Lyapunov stability. The Hamiltonian differs from the total energy in the important case of gyroscopic systems. Certain statements about mechanics will be made, and then theorems on mechanical stability will be stated.

Hamiltonian Systems

If the equations of motion of a mechanical system are written in Hamiltonian form, the result is [1]

$$p_i = -(\partial H/\partial q_i) + Q_i \qquad (i = 1,2,\ldots N)$$
$$\dot{q}_i = (\partial H/\partial p_i) \tag{1}$$

where $H(p,q)$ is the Hamiltonian function, assumed to be free of explicit dependence on time, the q_i are generalized coordinates of the problem, and the p_i are generalized momenta. Q_i is a generalized force, not derived from a potential function. The equations of motion (1) can be seen to imply the power balance relation

$$\dot{H} = P = \sum_{i=1}^{N} Q_i\,\dot{q}_i \tag{2}$$

where $P = \dot{H}$ is called the power. If $\dot{H}$ is calculated from the kinetic and potential energy expressions, it is

$$H = T_2 + U \tag{3}$$

where $T = T_2 + T_1 + T_0$, T_n is a homogeneous form of nth degree in the $\dot{q}_i$, and $U = V - T_0$. The proof of (3) is immediate from Euler's theorem on nth degree homogeneous forms.

It is important to observe that, if the total energy is defined as $E = T + V$, then

$$E - H = T_1 + 2T_0$$

This expression shows that there is a difference between E and H that is, in general, dependent on time.

Stability Theorems

Based upon the stability theory of Lyapunov[2,3] the definition of stability may be stated. It is assumed that the equilibrium point $[(\partial H/\partial p_i) = 0,\ (\partial H/\partial q_i) = 0]$ is $\mathbf{x} = 0$ where $\mathbf{x}(t) = (q_1,q_2,\ldots q_N,p_1,p_2\ldots p_N)$. If, given a system of differential equations (1), there exists an initial phase vector $\mathbf{x}(0)$ small enough $[\|\mathbf{x}(0)\| \leq \epsilon > 0]$ so that $\|\mathbf{x}(t)\|$ is within an arbitrary bound $[\|\mathbf{x}(t)\| \leq \delta > 0]$, then (1) is said to be stable. If, in addition to being stable, $\|\mathbf{x}\| \to 0$ as $t \to \infty$, then (1) is said to be asymptotically stable. Based upon this definition of stability (a more precise definition is stated in the references), we state the following important result.

Theorem

If for the autonomous mechanical systems described by (1) the power $P = \dot{H}$ is negative definite in a region S of the $\mathbf{x}$-space, then the motions are: 1) asymptotically stable if $H(p,q)$ is positive definite in S, or 2) unstable if $H(p,q)$ is sign variable or negative definite in S.

Part 1 of the theorem is proved using Lyapunov's theorem on asymptotic stability with H as a "testing function." Part 2 is proved using Lyapunov's theorem on instability with H as a "testing function."[4]

A corollary to the preceding theorem may be stated after observing that a necessary and sufficient condition for asymptotic stability is that H be positive definite in $\mathbf{x}$ if P is negative definite. Since (3) shows H to be the sum of T_2, a positive definite quadratic form in $\dot{q}_i$, and U to be a function of q_i, we see that U must be a positive definite function of q_i in order for H to be positive definite.

Corollary 1

If the hypothesis of the theorem is satisfied, then the motions are: 1) asymptotically stable if $U(q)$ is positive definite, or 2) unstable if $U(q)$ is nonpositive definite in the q_i.

Corollary 2

Corollary 2 is an important qualitative result. If the system obeys the hypothesis of the theorem, then its stability behavior cannot depend upon the magnitude or the analytical form of the power function. This result is true because the function U does not contain parameters associated with the sign of P. Testing for stability reduces to examining only U if P is negative. A very important corollary to the theorem is proved by Lefschetz and La Salle.[3] This corollary states that the theorem is true even if P is only negative semidefinite as long as there are no "decoupled motions."

Corollary 3

The condition of the theorem that P must be negative definite in $\mathbf{x}$ may be replaced by 1) $\dot{H} = P \leqq 0$ for all $\mathbf{x}$, and 2) $\dot{H} = P$ does not vanish identically (for all t) for motion not at the point $\mathbf{x} = 0$.

Results

The results of this note have been found to be important and useful in treating various space dynamics problems. It is imperative that we keep the fundamental distinction between E and H in mind in space problems involving rotating coordinates or cyclic variables. The results of this note with an expanded treatment and many applications will be found in Ref. 4.

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